current of 10.4 mA. What value should V_{DS} exceed?

5–27. An enhancement NMOS FET has $\beta = 0.5 \times 10^{-3}$ and $V_T = 2.5$ V. Find the value of I_D when (a) $V_{GS} = 6.14$ V, and (b) $V_{GS} = 0$ V.

5–28. An enhancement PMOS FET has $\beta = 0.5 \times 10^{-3}$ and $V_T = -2$ V. What is the value of V_{GS} when $I_D = 10.32$ mA?

5–29. An enhancement NMOS FET has the transfer characteristic shown in Figure 5–37.

    (a) Graphically determine V_{GS} when $I_D = 6.4$ mA.

    (b) Algebraically determine V_{GS} when $I_D = 6.4$ mA.

5–30. In the bias circuit of Figure 5–30, $R_1 = 2.2$ mΩ, $R_2 = 1$ mΩ, $V_{DD} = 28$ V, $R_D = 2.7$ kΩ, and $R_s = 600$ Ω. If $V_{GS} = 5.5$ V, find (a) I_D and (b) V_{DS}.

5–31. In the bias circuit of Figure 5-38, $R_1 = 470$ kΩ, $V_{DD} = 20$ V, $R_D = 1.5$ kΩ, $R_s = 220$ Ω, $I_D = 6$ mA, and $V_{GS} = 6$ V. Find (a) V_{DS} and (b) R_2.

5–32. The MOSFET in Figure 5–66 has $\beta = 0.62 \times 10^{-3}$ and $V_T = -2.4$ V. Algebraically determine the quiescent values of I_D, V_{GS}, and V_{DS}. Verify the validity of your results.
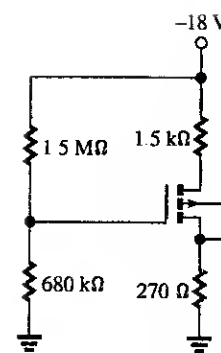

FIGURE 5–66   Exercise 5–32

SECTION 5–8
Integrated-Circuit MOSFETs

5–33. What does CMOS stand for? Why is it so named?

SECTION 5–9
VMOS and DMOS Transistors

5–34. List and briefly describe three advantages of VMOS technology.

5–35. What process is used to construct DMOS transistors? What is their primary application? What characteristics make them different from conventional MOSFETs?

SPICE EXERCISES

5–36. Verify the answers to Example 5–3 using SPICE analysis. BETA = .62SE−3, VTO = −4 for $V_{GS} = -1.5$ V. Why do the answers in Example 5–3 differ from the SPICE results?

5–37. Use SPICE to simulate the JFET chopper circuit shown in Figure 5–28. Use a 1–V p-p 1-kHz sinusoid for the input, a source resistance of 600 Ω, and a gate pulse of 0 to −5 V with a pulse width of 100 μs and a period of 200 μs. The BETA of the transistor is 7.5 E−4 and VTO = −4.

```
VS 5 0 SIN(0.5 1kHz)
RS 5 2 600
VG 3 0 PULSE(0 -5 0 1µs 1µs 100µs 200µs)
.MODEL NFET NJF BETA = 7.5E-4 VTO=-4
```

Provide a plot of the input sinusoid, the gate pulse, and the output waveform. What is the output waveform called?

CHAPTER 6

AMPLIFIER FUNDAMENTALS

▨ OUTLINE

OBJECTIVES

- Develop an understanding of the amplification function.
- Visualize an amplifier in its different components.
- Rationalize the concepts of voltage, current, and power gains.
- Perform circuit analysis using amplifier models.
- Understand the difference between single-ended and differential amplifiers.
- Recognize and analyze cascading amplifier configurations.

6–1 INTRODUCTION

Within an electronics context, the term amplifier refers to a circuit used for boosting electric signals in a prescribed manner. These signals will typically be generated by *transducers* such as microphones, tape heads, thermocouples, and electrochemical cells. They can also come from self-contained electronic components or instruments such as tape decks, CD players, pressure sensors, among many others.

Let us consider a basic stereo amplifier with a turntable (remember that?) connected to it. The signal stamped in the grooves of the vinyl record (disc) is picked up by a needle and a magnetic cartridge (also called magnetic pickup) and converted to a very small ac signal voltage that represents music and voice. The corresponding sound will eventually come from a pair of loud-speakers connected to the output of the amplifier. The few millivolts of signal produced by the turntable's magnetic pickup is not enough for producing a sound on the loudspeaker. The stereo amplifier provides the necessary boost-ing of the signal and delivers the appropriate voltage level to the speakers.

Signal amplification in the stereo amplifier is accomplished by two iden-tical multistage amplifiers which process the left and right signals that will eventually drive the speakers to produce sound. The initial stages boost the signal and provide additional processing such as tone control, balance, vol-ume, and other functions. This section of the stereo amplifier is called the *pre-amplifier*. The signal from the pre-amplifier, although several hundred times larger than that from the turntable, is not yet capable of driving a pair of speakers. It is only strong enough to drive a set of headphones. The stage that can finally drive a speaker is a *power amplifier* that can produce several volts across the speaker load, which is typically 4 Ω to 8 Ω, and produce sound at relatively high levels.

Other types of amplifiers used in fields such as industry and medicine can process different types of signals. In this chapter, we will talk about amplifiers in the general sense without regard to the types of signals they will process. We will also look at the different ways of representing an amplifier using equivalent electric circuits. In this manner, amplifier analysis can easily be done using basic circuit theory.

6–2 AMPLIFIER CHARACTERISTICS

Consider the block illustrated in Figure 6–1(a), which shows two pairs of terminals. It is common in network analysis to refer to those pairs of ter-minals as *ports*. In this context, the block is said to be a *two-port network*. For our discussion, let port 1 be the input port and port 2 the output port. If we apply a voltage across the input port, as illustrated in Figure 6–1(b), and obtain a larger voltage across the output port, then we regard the block as being a voltage amplifier. Additionally, if upon connecting a load resist-ance across the output port, as shown in Figure 6–1(c), the load or output current obtained is larger than the input current, then we say that the block is also a current amplifier. An amplifier block can amplify voltage, or cur-rent, or both. And as a consequence, it can also amplify power.

There are many types of amplifiers; some can amplify not only ac or sig-nal voltages but also dc voltages. Some others will produce sign inversion in addition to amplification. Amplifiers that are designed to produce large amounts of output current to drive low-impedance loads are regarded as power amplifiers. But there are several circuit parameters that are shared by all amplifiers, regardless of their type. These are

- input resistance,
- voltage gain,
- current gain,
- output resistance, and
- power gain.

Input Resistance

By definition, the input resistance of any network is the total equivalent resistance at its input terminals. In other words, it is the resistance "seen" by the input signal source when connected to the amplifier. Input resistance can also have different values depending on whether the input voltage or current is dc or ac. Furthermore, it can also vary with the frequency of the applied signal. But regardless of these facts, input resistance will always be defined as the ratio of input voltage to input current, as prescribed by Ohm's law. At this point, however, let us make the (possible) distinction between dc and ac input resistance, namely,
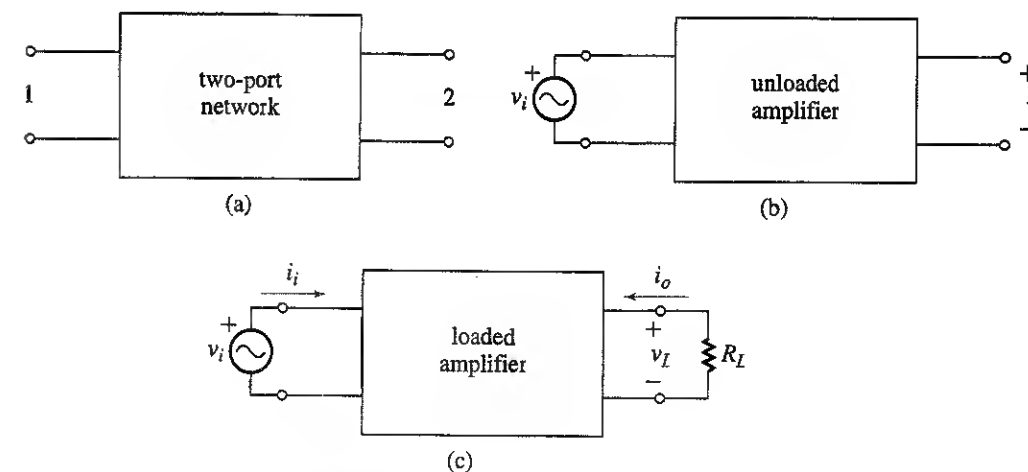

(a)

(b)

(c)

FIGURE 6–1 An amplifier pictured as a two-port network

$$R_i = V_i/I_i \quad \text{(dc input resistance)} \qquad (6\text{--}1)$$

and

$$r_i = v_i/i_i \quad \text{(ac input resistance)} \qquad (6\text{--}2)$$

where uppercase notation is used for dc quantities and lowercase notation for ac quantities.

Voltage Gain

Voltage gain is the voltage amplification factor from input to output. In words, output voltage equals input voltage *times* voltage gain. The voltage gain is therefore the *ratio* of output voltage to input voltage. Symbolized A_v, voltage gain is then defined by

$$A_v \equiv \frac{v_o}{v_i} \qquad (6\text{--}3)$$

Let us stress the fact that voltage gain is the ratio, and not the difference, between output and input voltages. Also, if the amplifier produces sign inversion, the gain is said to be inverting and its value will be preceded by a negative sign.

Current Gain

Like voltage gain, current gain is an amplification factor defined as the ratio of output current to input current. Obviously, in order for an output current to actually take place, there must be a path for it, typically a load resistance, as seen in Figure 6–1(c). If the path for the output current is a short circuit across the output terminals, the current gain would be regarded as *short-circuit current gain*. Current gain, symbolized by A_i, is therefore defined as

$$A_i \equiv \frac{i_o}{i_i} \qquad (6\text{--}4)$$

We will specify the output current as "entering" the positive output terminal. Note that this is an arbitrary reference and does not imply the actual direction of the output current. If the output current actually enters the positive output terminal when the input current enters (assumed reference) the positive input terminal, we say that the current gain is positive or noninverting. It is important to clarify that the reference direction for the output current is not universal and that technical literature and textbooks use it in either way. As long as there is consistency, the results will be identical one way or the other.

Output Resistance

The amplifier's output resistance is the resistance "seen" by the load resistance when "looking back" into the output terminals. Output resistance can also be interpreted as being the Thévenin resistance at the output terminals. This way, the output resistance can be obtained by driving the output port with an external source to obtain the v/i ratio at the output terminals. This has to be done with the input source deactivated, that is, replaced by a short circuit if it is a voltage source or by an open circuit if a current source, but leaving the source resistance, if any, in the circuit. The output resistance in this case is defined by

$$r_o \equiv \frac{v_{ext}}{i_{ext}} \qquad (6\text{--}5)$$

where v_{ext} is the applied external voltage and i_{ext} is the resulting current.

Alternately, the Thévenin or output resistance can be obtained by driving the block with its normal input source to obtain the open-circuit voltage v_{oc} and short-circuit current i_{sc} at the output terminals. The output resistance can then be expressed as

$$r_o \equiv \frac{v_{oc}}{i_{sc}} \qquad (6\text{--}6)$$

Incidentally, you may recall that the short-circuit current, by definition, is the Norton current I_N from Norton's theorem and that the ratio V_{Th}/I_N is the Thévenin (or Norton) resistance.

Power Gain

Power gain, symbolized A_p, is defined as the ratio of output power to input power, that is,

$$A_p \equiv \frac{p_o}{p_i} \qquad (6\text{--}7)$$

But since $p_i = v_i i_i$ and $p_o = v_o i_o$, current gain can also be expressed as the product of the voltage and current gains, namely,

$$A_p = \frac{v_o i_o}{v_i i_i} = A_v A_i \qquad (6\text{--}8)$$

Input power can also be computed using any of the other familiar forms:

$$p_i = i_i^2 r_i \quad \text{or} \quad p_i = v_i^2/r_i \qquad (6\text{--}9)$$

An important point: Voltage and current gain, as well as input and output resistance, can be determined using rms, peak, and peak-to-peak units, as long as consistency is observed. However, power calculations using the standard formulas cited above must be carried out using rms units for both voltage and current in order to obtain correct results. Alternatively, the following modified formulas can be used for peak and peak-to-peak units.

$$p = \frac{v_p i_p}{2} = \frac{i_p^2 R}{2} = \frac{v_p^2}{2R} \quad \text{(peak units)} \qquad (6\text{--}10)$$

$$p = \frac{v_{pp} i_{pp}}{8} = \frac{i_{pp}^2 R}{8} = \frac{v_{pp}^2}{8R} \quad \text{(peak-to-peak units)} \qquad (6\text{--}11)$$

Unless it is necessary to emphasize that we are using a particular form of voltage (peak, p-p, rms), we will simply use lowercase notation for ac quantities with no units in particular. Again, as long as we are consistent, all calculations will be correct.

The Voltage Gain Formula

Because input voltage and input current in an amplifier are related by the input resistance, and the output voltage is the product of output current and load resistance, we can obtain an expression for the voltage gain as a function of the current gain as follows:

Observing Figure 6–1(c), input and output voltages are

$$v_i = i_i r_i \quad \text{and} \quad v_L = -i_o R_L$$

The *loaded* voltage gain is then

$$A_v = \frac{v_L}{v_i} = \frac{-i_o R_L}{i_i r_i}$$

which can be written as

$$A_v = -A_i \frac{R_L}{r_i} \tag{6-12}$$

Equation 6–12 is a general expression for the loaded voltage gain that will always hold true regardless of the type or structure of the amplifier, provided both input and output currents are defined entering the positive terminals. In case the output current is defined leaving the positive output terminal, equation 6–12 would have to be written as a positive relationship. Later in the chapter, we will make use of this expression to perform gain calculations and for developing other important formulas.

EXAMPLE 6-1

Figure 6-2 shows the conventional symbol for an amplifier: a triangular block with output at the vertex. As shown in the figure, the input voltage to the amplifier is $v_i(t) = 0.7 + 0.008 \sin 10^3 t$ V. The amplifier has an ac current gain of 80. If the input current is $i_i(t) = 2.8 \times 10^{-5} + 4 \times 10^{-6} \sin 10^3 t$ A, and the ac component of the output voltage is 0.4 V rms, find (1) A_v, (2) R_i, (3) r_i, (4) i_o (rms), (5) R_L, and (6) A_p.

Solution

1. v_i (rms) = 0.707(0.008 V-pk) = 5.66×10^{-3} V rms

$$A_v = \frac{v_o(\text{rms})}{v_i(\text{rms})} = \frac{0.4\ \text{V}}{5.66 \times 10^{-3}\ \text{V}} = 70.7$$

2. The dc input resistance is the ratio of the dc component of the input voltage to the dc component of the input current:

$$R_i = \frac{V_i}{I_i} = \frac{0.7\ \text{V}}{2.8 \times 10^{-3}\ \text{A}} = 25\ \text{k}\Omega$$

3. The ac input resistance is the ratio of the ac components of the input voltage and current:

$$r_i = \frac{v_i}{i_i} = \frac{0.008\ \text{V-pk}}{4 \times 10^{-6}\ \text{A-pk}} = 2\ \text{k}\Omega$$

4. i_o (rms) = $A_i i_i$ (rms) = 80(0.707)(4 × 10⁻⁶ A-pk) = 0.226 mA rms

5. $R_L = \frac{v_o(\text{rms})}{i_o(\text{rms})} = \frac{0.4\ \text{V}}{0.226 \times 10^{-3}\ \text{A}} = 1770\ \Omega$

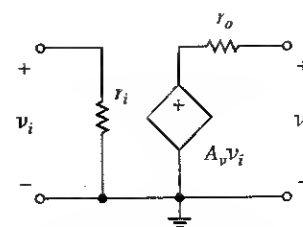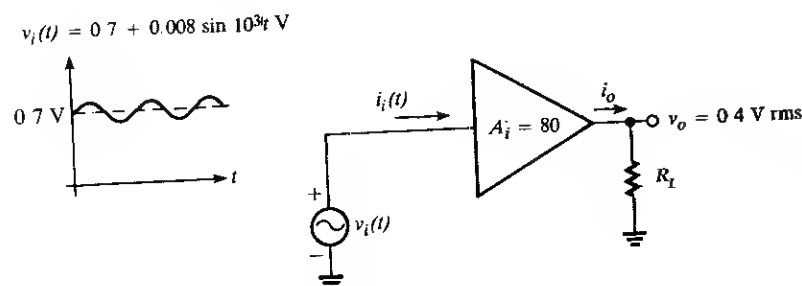FIGURE 6-2 (Example 6-1) $v_i(t) = 0.7 + 0.008 \sin 10^3 t$ V

FIGURE 6-3 Amplifier model with a voltage-controlled voltage source

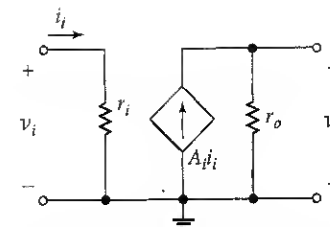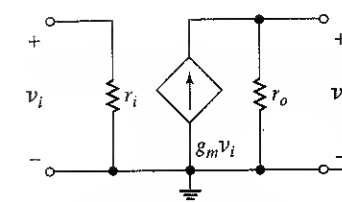FIGURE 6-4 Amplifier model with a current-controlled current source

FIGURE 6-5 Amplifier model with a voltage-controlled current source

6. $p_i = \frac{v_i^2(\text{rms})}{r_i} = \frac{(5.66 \times 10^{-3}\ \text{V})^2}{2 \times 10^3\ \Omega} = 1.6 \times 10^{-8}$ W

$p_o = \frac{v_o^2(\text{rms})}{r_o} = \frac{(0.4\ \text{V})^2}{1770\ \Omega} = 9.04 \times 10^{-5}$ W

$A_p = \frac{p_o}{p_i} = \frac{9.04 \times 10^{-5}\ \text{W}}{1.6 \times 10^{-8}\ \text{W}} = 5650$

Note that the power gain can also be computed in this example as the product of the voltage and current gains: $A_p = A_v A_i = (70.7)80 = 5656$. The small difference between the two results is due to roundoff error.

6-3 AMPLIFIER MODELS

Now that we have a good idea of how an amplifier behaves with respect to input and output terminals, let us look at equivalent circuits, called models, that emulate the internal behavior of an amplifier. We will use special kinds of sources called dependent, or controlled, voltage and current sources. Additionally, we will adopt the standard symbol for dependent sources: a tilted square.
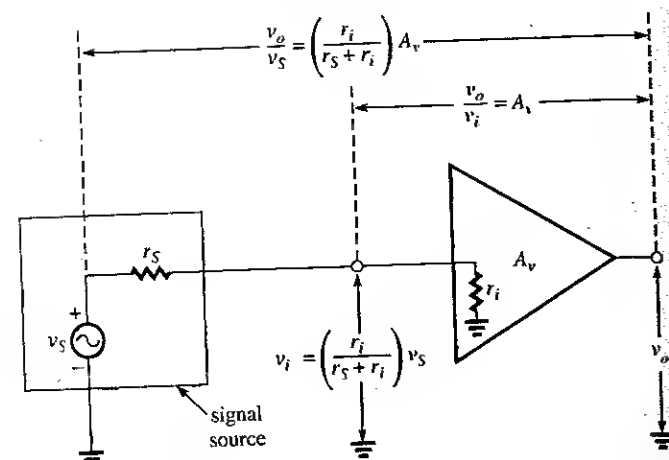
The model in Figure 6-3 employs a *voltage-controlled voltage source* (VCVS) on the output circuit. Notice that the output resistance, r_o, appears in series with the VCVS. Notice also the labeling of the VCVS: $A_v v_i$; it means that the voltage generated by the VCVS is the input voltage times the dimensionless factor A_v. The input voltage is said to be the controlling variable. As you can see, this model is based on voltage gain.

The model shown in Figure 6-4 makes use of a current-controlled current source (CCCS) and a parallel output resistance, that is, a Norton equivalent circuit. Note that this model is based on current gain because the current generated by the CCCS is the input current times the dimensionless factor A_i.

The last model we will consider is shown in Figure 6-5. It employs a *voltage-controlled current source* (VCCS) on the output. The VCCS generates a current that is proportional to the input voltage. The factor g_m is called the *transconductance* of the amplifier and has units of siemens.

These models are used with all types of amplifiers, whether they employ vacuum tubes or semiconductor devices such as transistors. Furthermore, they can be used in amplifiers based on a single electronic device as well as in those built with multiple devices. In a later chapter we will study *operational amplifiers*, which are multitransistor structures in integrated circuit form. Operational amplifiers are the basic and most important building blocks in all modern amplifier designs.

FIGURE 6–6 r_S and r_i form a voltage divider across the amplifier input. The voltage gain from source to output is reduced by the factor $r_i/(r_S + r_i)$.

Source Resistance

Every signal source has internal resistance (its Thévenin equivalent resistance), which we will refer to as *source resistance*, r_S. When a signal source is connected to the input of an amplifier, the source resistance is in series with the input resistance, r_i, of the amplifier. Notice in Figure 6–6 that r_S and r_i form a voltage divider across the input to the amplifier. The input voltage at the amplifier is

$$v_i = v_S\left(\frac{r_i}{r_S + r_i}\right) \qquad (6\text{–}13)$$

Now,

$$v_o = A_v v_i = A_v v_S\left(\frac{r_i}{r_S + r_i}\right)$$

So

$$\frac{v_o}{v_S} = A_v\left(\frac{r_i}{r_S + r_i}\right) \qquad (6\text{–}14)$$

Equation 6–14 shows that the overall voltage gain *between source voltage and amplifier output*, v_o/v_S, equals the amplifer voltage gain *reduced* by the factor $r_i/(r_S + r_i)$.

If r_i is much larger than r_S, then $r_i/(r_S + r_i) \approx 1$, so there is little reduction in the overall voltage gain caused by the voltage-divider effect. It is therefore desirable, in general, for a voltage amplifier to have as large an input resistance as possible.

On the other hand, if current amplification is desired, the amplifier should have as *small* an input resistance as possible. When r_i is small, the majority of the current generated at the signal source will be delivered to the amplifier input. This fact is illustrated in Figure 6–7, where the signal is a current source.

As shown in Figure 6–7, the current delivered to the amplifier input is the source current i_S reduced by the factor $r_S/(r_S + r_i)$. Therefore, r_i should be much less than r_S to make the quantity $r_S/(r_S + r_i)$ close to 1. The overall current gain from source to output is

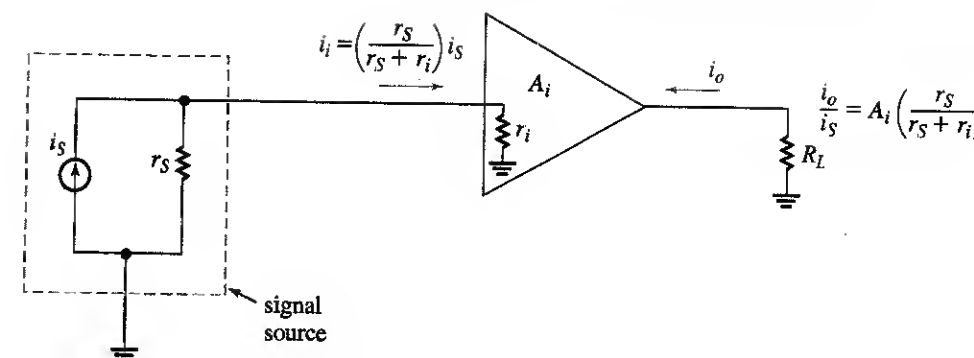$$\frac{i_o}{i_S} = A_i\left(\frac{r_S}{r_S + r_i}\right) \qquad (6\text{–}15)$$

FIGURE 6–7 A current amplifier should have a small input resistance in order to make the quantity $r_S/(r_S + r_i)$ close to 1.

The amplifier shown in Figure 6–8 has $A_i = 10$ and $r_i = 10$ kΩ. It is driven by a signal source that has source resistance r_S. Find the overall voltage gain from source to output when (1) $r_S = 1$ kΩ and (2) $r_S = 10$ kΩ. Note: Observe the orientation of i_o.

Solution

1. $r_S = 1$ kΩ $(r_S = 0.1\ r_i)$ $A_v = A_i\dfrac{R_L}{r_i} = 10\left(\dfrac{2\ \text{k}}{1\ \text{k}}\right) = 20$

$$\frac{v_o}{v_S} = A_v\left(\frac{r_i}{r_S + r_i}\right) = 20\left(\frac{10\ \text{k}}{1\ \text{k} + 10\ \text{k}}\right) = 18.2$$

2. $r_S = 10$ kΩ $(r_i = 0.1 r_S)$

$$\frac{v_o}{v_S} = 20\left(\frac{10\ \text{k}}{10\ \text{k} + 10\ \text{k}}\right) = 10$$

This example shows that when $r_S = 0.1\ r_i$, the voltage gain is reduced by about 10%; when $r_i = r_S$, the voltage is reduced by 50%.

Load Resistance

An ac amplifier is always used to supply voltage, current, and/or power to some kind of *load* connected to its output. The load may be a speaker, an antenna, a siren, an indicating instrument, an electric motor, or any one of a large number of other useful devices. Often the load is the input to another amplifier. Amplifier performance is analyzed by representing its load as an equivalent load resistance (or impedance). When a load resistance R_L is connected to the output of an amplifier, there is again a voltage division
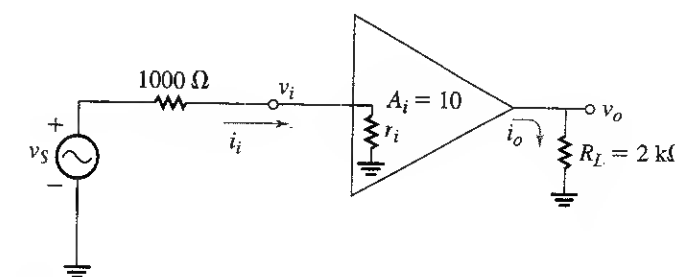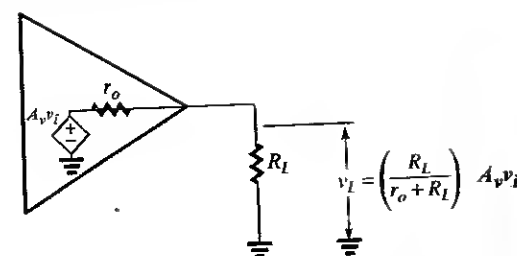
FIGURE 6–8 (Example 6–2).

FIGURE 6-9   The output voltage from an
ac amplifier divides between r_o and the load
resistance R_L

between the output resistance of the amplifier and the load. Figure 6-9
shows a Thévenin equivalent circuit of the output of an ac amplifier in which
the output circuit is modeled by a VCVS in series with r_o. As can be seen in
the figure, the load voltage v_L is determined by

$$v_L = \left(\frac{R_L}{r_o + R_L} \right) A_v v_i \tag{6-16}$$

Note that $A_v v_i$ should be regarded as the open-circuit (or unloaded) output
voltage. For a voltage amplifier, r_o should be much smaller than R_L in order
to maximize the portion of $A_v v_i$ that appears across the load.

When the effects of both r_S and R_L are taken into account, the overall volt-
age gain A_{vs} from source to load becomes

$$A_{vs} = \frac{v_L}{v_S} = A_v \left(\frac{r_i}{r_S + r_i} \right) \left(\frac{R_L}{r_o + R_L} \right) \tag{6-17}$$

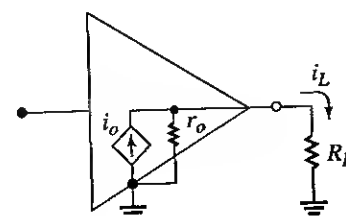where A_v is the unloaded voltage gain.

If a signal source has fixed resistance r_S, then from the *maximum
power transfer theorem*, maximum power is transferred from the source to an
amplifier when $r_i = r_S$. Similarly, if the amplifier has fixed output resistance
r_o, then maximum power is transferred from the amplifier to a load when
$R_L = r_o$. The amplifier is said to be *matched* to its source when $r_i = r_S$ and
matched to its load when $R_L = r_o$. (We should also note that if the values of r_S
and r_o can be controlled, maximum power transfer occurs when $r_S = 0$ and
$r_o = 0$, irrespective of the values of r_i and R_L.)

As we have seen, the output of an amplifier can also be modeled using a
current source in parallel with the output resistance, as shown in Figure 6-10.
This is the Norton equivalent, which can be very useful for devices that are
more current oriented such as transistors.

Loading effects under this situation are opposite to those for the
Thévenin model. It is apparent that in this case more current will flow
through the load if R_L is much less than r_o. Therefore, a well-designed current
amplifier should have relatively large output resistance, as opposed to a volt-
age amplifier, whose output resistance should be as small as possible. The
direction shown for the current depends on the particular device and con-
figuration used in the amplifier. The open-circuit output voltage is clearly

$$v_o = i_o r_o \tag{6-18}$$

FIGURE 6-10   Norton equivalent
of the output. The current i_o can be
a function of v_i or i_i.

where i_o can either be $g_m v_i$ or $A_i i_i$. With the load resistance connected, the
load voltage v_L can be obtained by multiplying i_o times the parallel combi-
nation $r_o \| R_L$, that is,

$$v_L = i_o(r_o \| R_L) \tag{6-19}$$

or by using voltage division on the open-circuit output voltage $(i_o r_o)$ in series
with the output resistance r_o, that is, "Thévenizing" the parallel circuit. Thus,

$$v_L = (i_o r_o) \frac{R_L}{R_L + r_o} \tag{6-20}$$

which is equivalent to (6-19).

Alternatively, we could have performed current division to obtain i_L in
terms of i_o and then multiplied by R_L to obtain the load voltage v_L, that is,

$$v_L = i_o \left(\frac{r_o}{r_o + R_L} \right) R_L$$

which yields (6-19) or (6-20).

Inverting and Noninverting Amplification

We had already learned that when an amplifier produces sign inversion it is
regarded as an inverting amplifier. The models of Figures 6-3, 6-4, and 6-5
are all of the noninverting type, because a positive input voltage produces a
positive output voltage. An inverting model can be obtained by either flip-
ping over the dependent source or by indicating a negative voltage or current
gain, or a negative transconductance factor.

As an example, Figure 6-11 shows an inverting amplifier modeled with
the transconductance function g_m. Note the orientation of the VCCS, which
produces a negative voltage across the output terminals, according to the
indicated reference for v_o.

Let us obtain the overall voltage gain of this circuit when driven by a volt-
age source with a characteristic resistance r_S.

First, the voltage across the input, using voltage division is
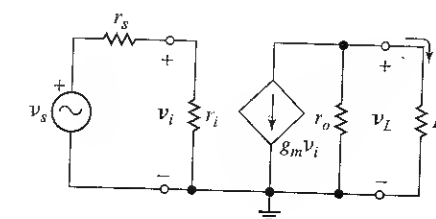
$$v_i = v_S \frac{r_i}{r_S + r_i} \tag{6-21}$$

The loaded output voltage is clearly

$$v_L = -g_m v_i (r_o \| R_L) \tag{6-22}$$

that yields

$$v_L = -g_m v_S \frac{r_i}{r_S + r_i} (r_o \| R_L) \tag{6-23}$$

FIGURE 6-11   Inverting amplifier
with the transconductance model

from which we solve for the overall gain A_{vs}:

$$A_{vs} = \frac{v_L}{v_S} = -g_m(r_o\|R_L)\frac{r_i}{r_S + r_i} \qquad (6\text{-}24)$$

To further illustrate analysis with models, let us also find the loaded current gain from the input source to the load resistance, that is, i_L/i_S. Using the current-division rule on the parallel output circuit we can write

$$i_L = -\frac{r_o}{r_o + R_L}g_m v_i \qquad (6\text{-}25)$$

But $v_i = i_S r_i$; therefore,

$$\frac{i_L}{i_S} = -g_m r_i \frac{r_o}{r_o + R_L} \qquad (6\text{-}26)$$

EXAMPLE 6-3

The amplifier shown in Figure 6–11 has the following parameters: $r_i = 2.4$ kΩ, $g_m = 48$ mS, $r_o = 75$ kΩ. The amplifier has an external resistor $R_1 = 12$ kΩ connected across the input terminals, and the load resistance is 10 kΩ. If the input source resistance is 1 kΩ, find:

1. The overall current gain i_L/i_S.
2. The overall voltage gain A_{vs}.
3. The power gain from the input to the load resistance.

Solution

1. Using (6–26) and taking into account the current-division effect at the input, we write

$$\frac{i_L}{i_S} = -g_m r_i \left(\frac{r_o}{r_o + R_L}\right)\left(\frac{R_1}{R_1 + r_i}\right) = -84.7$$

2. To take into account R_1 in the calculation of the overall voltage gain, we simply put it in parallel with r_i in (6–24):

$$A_{vs} = -g_m(r_o\|R_L)\frac{r_i\|R_1}{r_s + r_i\|R_1} = -282.4$$

3. The power gain from input to load can be obtained as the product of the voltage and current gains, provided the voltage gain is with respect to v_i. From (6–22), this gain is

$$\frac{v_L}{v_i} = -g_m(r_o\|R_L) = -423.5$$

and the power gain is

$$A_p = \frac{i_L}{i_S} \times \frac{v_L}{v_i} = 3.587 \times 10^4$$

Differential Amplifiers

The models of Figures 6–3, 6–4, and 6–5 are referred to as single-ended amplifiers, meaning that both input and output voltages are referenced to ground. Another form, called *differential amplifier*, employs a double-ended
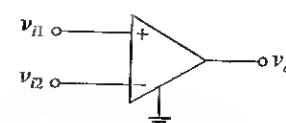
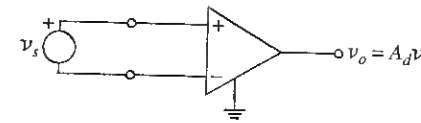FIGURE 6–12 Basic differential amplifier

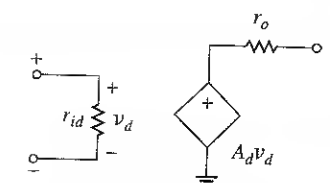FIGURE 6–13 An input signal connected in the differential mode

FIGURE 6–14 Basic differential amplifier model

input. The output can either be single- or double-ended. A double-ended or differential input consists of two terminals as shown in Figure 6–12.

The basic relationship between input and output in a differential amplifier is

$$v_o = A_d(v_{i1} - v_{i2}) \qquad (6\text{-}27)$$

where A_d is the differential voltage gain of the amplifier. Note that the output voltage is the differential gain times the *difference* between the two input voltages. This difference is called the differential input voltage; however, in most practical situations there will only be one input signal which will be connected in a "floating" mode as shown in Figure 6–13.

A very common application of differential amplifiers is in microphone signal amplification. Due to the normally long cables associated with microphones, noise and interference will be picked up by the conductors. If a pair of twisted wires with an overall grounded shield is used for connecting the microphone to the differential amplifier, each of the conductors of the twisted pair will carry identical noise voltages, thereby cancelling at the differential amplifier. The signal voltage across the twisted pair appears as a differential input voltage and will be amplified by the differential gain. In a later chapter we will look at noise calculations in differential amplifiers.

An important issue regarding differential amplifiers is that, if needed, they can be used as conventional (signal-ended) inverting or noninverting amplifiers. A noninverting amplifier can be obtained by applying the input signal to the noninverting (+) input while connecting the inverting (−) input to ground. Similarly, an inverting amplifier is obtained by using the inverting (−) input terminal and grounding the noninverting (+) one.

A basic model for a differential amplifier is shown in Figure 6–14. The single-ended output includes the output resistance r_o.

In certain applications, there is need for the output to be differential as well. Figure 6–15 shows a differential amplifier with double-ended output. In this case the input-output relationship is expressed as

$$v_{o1} - v_{o2} = A_d(v_{i1} - v_{i2}) \qquad (6\text{-}28)$$

where the output signal is also a differential voltage that appears across the two output terminals. Double-ended outputs are useful when there is need for sending an output signal from a differential amplifier, through a long cable, to another differential amplifier. This way, the noise-cancelling property of differential amplifiers is preserved throughout the amplification process.

FIGURE 6–15 A differential amplifier with double-ended output

6–4   MULTISTAGE AMPLIFIERS

In many applications, a single amplifier cannot furnish all the gain that is required to drive a particular kind of load. For example, a speaker represents a
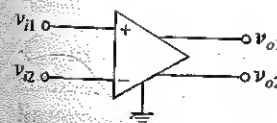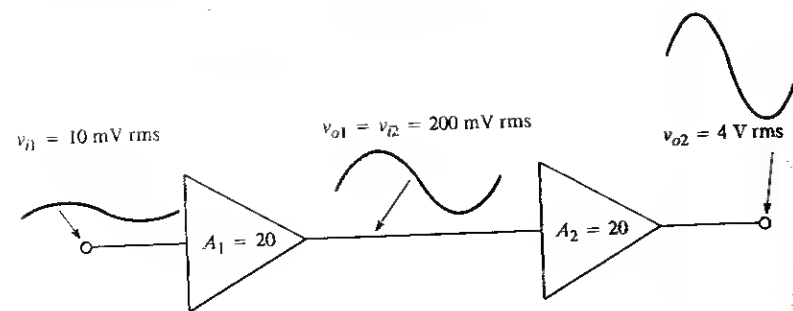
FIGURE 6-16   Two amplifier
stages connected in cascade

$v_{i1} = 10 \text{ mV rms}$ $v_{o1} = v_{i2} = 200 \text{ mV rms}$ $v_{o2} = 4 \text{ V rms}$

$A_1 = 20$ $A_2 = 20$

"heavy" load in an audio amplifier system, and several amplifier stages may be required to "boost" a signal originating at a microphone or magnetic tape head to a level sufficient to provide a large amount of power to the speaker. We hear of *preamplifiers, power amplifiers,* and o*utput amplifiers,* all of which constitute stages of amplification in such a system. Actually, each of these components may itself consist of a number of individual transistor amplifier stages. Amplifiers that create voltage, current, and/or power gain through the use of two or more stages are called multistage amplifiers.

When the output of one amplifier stage is connected to the input of another, the amplifier stages are said to be in *cascade.* Figure 6-16 shows two stages connected in cascade. To illustrate how the overall voltage gain of the combination is computed, let us assume that the input to the first stage is 10 mV rms and that the voltage gain of each stage is $A_1 = A_2 = 20$, as shown in the figure. The output of the first stage is $A_1 v_{i1} = 20$ (10 mV rms) = 200 mV rms. Thus, the input to the second stage is 200 mV rms. The output of the second stage is, therefore, $A_2 v_{i2} = 20$(200 mV rms) = 4 V rms. Therefore, the overall voltage gain is

$$A_v = \frac{v_{o2}}{v_{i1}} = \frac{4 \text{ V rms}}{10 \text{ mV rms}} = 400$$

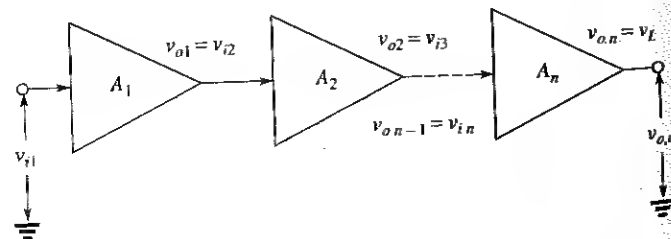Notice that $A_v = A_1 A_2 = (20)(20) = 400$.

Figure 6-17 shows an arbitrary number (n) of stages connected in cascade. Note that the output of each stage is the input to the succeeding one ($v_{o1} = v_{i2}, v_{o2} = v_{i3}$, etc.) We will derive an expression for the overall voltage gain $v_{o,n}/v_{i1}$ in terms of the individual stage gains $A_1, A_2, \ldots, A_n$. We assume that each stage gain $A_1, A_2 \ldots, A_n$ is the value of the voltage gain between input and output of a stage *with all other stages connected* (more about that important assumption later).

By definition,

$$v_{o1} = A_1 v_{i1} \qquad (6\text{--}29)$$

Also,

$$v_{o2} = A_2 v_{i2} = A_2 v_{o1} \qquad (6\text{--}30)$$

FIGURE 6-17 n amplifier stages
connected in cascade. The output
voltage of each stage is the input
voltage to the next stage

$v_{o1} = v_{i2}$ $v_{o2} = v_{i3}$ $v_{o,n} = v_L$

A_1 A_2 A_n

v_{i1} $v_{on-1} = v_{in}$ $v_{o,n}$

Substituting v_{o1} from (6-29) into (6-30) gives

$$v_{o2} = (A_2 A_1) v_{i1} \qquad (6\text{--}31)$$

Similarly,

$$v_{o3} = A_3 v_{i3} = A_3 v_{o2}$$

and, from (6-31),

$$v_{o3} = (A_3 A_2 A_1) v_{i1}$$

Continuing in this manner, we eventually find

$$v_{o,n} = v_L = (A_n A_{n-1} \ldots A_2 A_1) v_{i1}$$

Therefore,

$$\frac{v_{o,n}}{v_{i1}} = A_n A_{n-1} \ldots A_2 A_1 \qquad (6\text{--}32)$$

Equation 6-32 shows that the overall voltage gain of n cascaded stages is the *product* of the individual stage gains (not the sum!). In general, any one or more of the stage gains can be negative, signifying, as usual, that the stage causes a 180° phase inversion. It follows from equation 6-32 that the cascaded amplifiers will cause the output of the last stage ($v_{o,n}$) to be out of phase with the input to the first stage (v_{i1}) if there is an *odd* number of inverting stages, and will cause $v_{o,n}$ to be in phase with v_{i1} if there is an even (or zero) number of inversions.

Our derivation of equation 6-32 did not include the effect of source or load resistance on the overall voltage gain. Source resistance r_S causes the usual voltage division to take place at the input to the first stage, and load resistance r_L causes voltage division to occur between r_L and the output resistance of the last stage. Under those circumstances, the overall voltage gain between load and signal source becomes

$$\frac{v_L}{v_S} = \left(\frac{r_{i1}}{r_S + r_{i1}}\right) A_n A_{n-1} \ldots A_2 A_1 \left(\frac{r_L}{r_{o,n} + r_L}\right) \qquad (6\text{--}33)$$

where r_{i1} = input resistance to first stage and $r_{o,n}$ = output resistance of last stage.
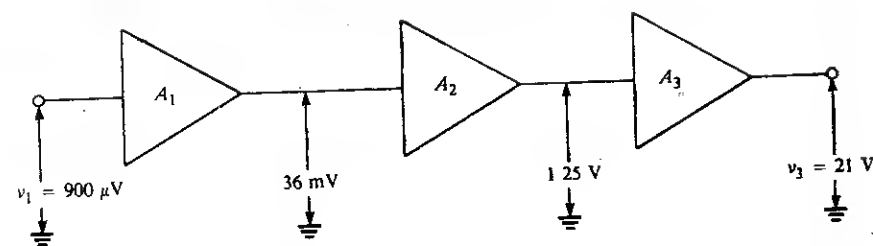
EXAMPLE 6-4

Figure 6-18 shows a three-stage amplifier and the ac rms voltages at several points in the amplifier. Note that v_1 is the input voltage delivered by a signal source having zero resistance and that v_3 is the output voltage with no load connected.

1. Find the voltage gain of each stage and the overall voltage gain v_3/v_1.
2. Find the overall voltage gain v_L/v_S when the multistage amplifier is driven by a signal source having resistance 2000 Ω and the load is 25 Ω. Stage 1 has input resistance 1 kΩ and stage 3 has output resistance 50 Ω.
3. What is the power gain under the conditions of (2) (measured between the input to the first stage and the load)?
4. What is the overall current gain i_L/i_1 under the conditions of (2)?

Solution

1. $A_1 = (36 \text{ mV})/(900 \text{ μV}) = 40$
   $A_2 = (1.25 \text{ V})/(36 \text{ mV}) = 34.722$
   $A_3 = (21 \text{ V})/(1.25 \text{ V}) = 16.8$

FIGURE 6–18   (Example 6–4)

$v_3/v_1 = A_1 A_2 A_3 = (40)(34\,722)(16.8) = 23{,}333$

Note that the product of the voltage gains equals the overall voltage gain, which, in this example, can also be calculated directly: $v_3/v_1 = (21\text{ V})/(900\ \mu\text{V}) = 23{,}333.$

2. From equation 6–33,

$$\frac{v_L}{v_S} = \left(\frac{1000}{2000 + 1000}\right)(23{,}333)\left(\frac{25}{50 + 25}\right) = 2592.5$$

3. When a signal-source resistance of 2000 Ω is inserted in series with the input, v_1 becomes

$$v_1 = \left(\frac{1000}{2000 + 1000}\right)(900\ \mu\text{V}) = 300\ \mu\text{V}$$

The input power is, therefore,

$$P_i = \frac{v_1^2}{r_{i1}} = \frac{(300 \times 10^{-6})^2}{1000} = 90\text{ pW}$$

The voltage across the 25-Ω load is then

$$v_L = v_1(A_1 A_2 A_3)\left(\frac{R_L}{r_{o3} + R_L}\right)$$

$$= (300\ \mu\text{V})(40)(34.722)(16.8)\left(\frac{25}{50 + 25}\right) = 2.33\text{ V}$$

The output power developed across the load resistance is, therefore,

$$P_o = \frac{v_L^2}{R_L} = \frac{(2.33)^2}{25} = 0.217\text{ W} = 217\text{ mW}$$

Finally,

$$A_p = \frac{217\text{ mW}}{90\text{ pW}} = 2.41 \times 10^9$$

4. Recall that $A_p = A_v A_i.$

The voltage gain between the input to the first stage and the load is

$$A_v = \frac{v_L}{v_1} = \frac{2.33\text{ V}}{300\ \mu\text{V}} = 7766$$

Therefore,

$$A_i = \frac{A_p}{A_v} = \frac{2.41 \times 10^9}{7766} = 3.1 \times 10^5$$

Interstage Loading

It is important to remember that the gain equations we have derived are based on the *in-circuit* values of $A_1, A_2, \ldots$, that is, on the stage gains that result when all other stages are connected. Thus, we have assumed that each value of stage gain takes into account the loading the stage causes on the previous stage and the loading presented to it by the next stage (except we assumed that A_1 did not include loading by r_S and A_n did not include loading by r_L). If we know the *open-circuit* (unloaded) voltage gain of each stage and its input and output resistances, we can calculate the overall gain by taking into account the loading effects of each stage on another. Theoretically, the load presented to a given stage may depend on *all* of the succeeding stages lying to its right, since the input resistance of any one stage depends on its output load resistance, which in turn is the input resistance to the next stage, and so forth. In practice, we can usually ignore this cumulative loading effect of stages beyond the one immediately connected to a given stage, or assume that the input resistance that represents the load of one stage to a preceding one is given for the condition that all succeeding stages are connected.

To illustrate the ideas we have just discussed, Figure 6–19 shows a three-stage amplifier for which the individual open-circuit voltage gains A_{o1}, A_{o2}, and A_{o3} are assumed to be known, as well as the input and output resistances of each stage. From the voltage division that occurs at each node in the system, it is apparent that the following relations hold:

$$v_1 = \left(\frac{r_{i1}}{r_S + r_{i1}}\right)v_S$$

$$v_2 = A_{o1}v_1\left(\frac{r_{i2}}{r_{o1} + r_{i2}}\right)$$

$$v_3 = A_{o2}v_2\left(\frac{r_{i3}}{r_{o2} + r_{i3}}\right)$$
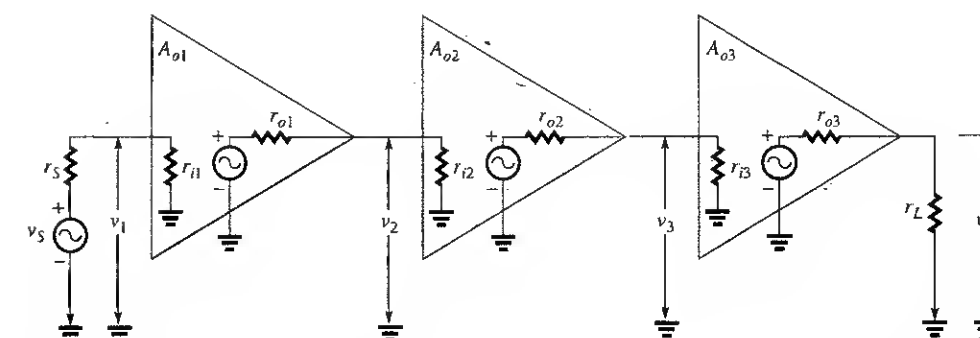
$$v_L = A_{o3}v_3\left(\frac{r_L}{r_{o3} + r_L}\right)$$

FIGURE 6–19   A three-stage amplifier. A_{o1}, A_{o2}, and A_{o3} are the open-circuit (unloaded) voltage gains of the respective stages.

Combining these relations leads to

$$\frac{v_L}{v_S} = \left(\frac{r_{i1}}{r_S + r_{i1}}\right) A_{o1}\left(\frac{r_{i2}}{r_{o1} + r_{i2}}\right) A_{o2}\left(\frac{r_{i3}}{r_{o2} + r_{i3}}\right) A_{o3}\left(\frac{r_L}{r_{o3} + r_L}\right) \quad (6\text{-}34)$$

As might be expected, equation 6–34 shows that the overall voltage gain of the multistage amplifier is the product of the open-circuit stage gains multiplied by the voltage-division ratios that account for the loading of each stage. Notice that a *single* voltage-division ratio accounts for the loading between any pair of stages. In other words, it is *not* correct to compute loading effects twice: once by regarding an input resistance as the load on a previous stage and again by regarding the output resistance of that previous stage as the source resistance for the next stage.

In amplifiers modeled with an output current source (CCCS or VCCS), the interstage loading is manifested as current division between the output resistance of one stage and the input resistance of the following stage. Figure 6–20 illustrates two cascaded amplifier stages. Let us analyze the circuit to find the overall transconductance i_L/v_S and the overall voltage gain v_L/v_S.

Starting on the input side, we can write an expression for the input current as

$$i_{i1} = \frac{v_S}{r_S + r_{i1}}$$

Using the current division rule, the input current to the second stage can be expressed as

$$i_{i2} = \frac{r_{o1}}{r_{o1} + r_{i2}} A_{i1} i_{i1}$$

$$= \frac{r_{o1}}{r_{o1} + r_{i2}} A_{i1} \frac{v_S}{r_S + r_{i1}}$$

Similarly, the load current is

$$i_L = \frac{r_{o2}}{r_{o2} + R_L} A_{i2} i_{i2}$$

$$= \frac{r_{o2}}{r_{o2} + R_L} A_{i2} \frac{r_{o1}}{r_{o1} + r_{i2}} A_{i1} \frac{v_S}{r_S + r_{i1}}$$

which yields the overall transconductance in terms of current gains:

$$\frac{i_L}{v_S} = \frac{A_{i1} A_{i2}}{r_S + r_{i1}} \left(\frac{r_{o1}}{r_{o1} + r_{i2}}\right)\left(\frac{r_{o2}}{r_{o2} + R_L}\right) \quad (6\text{-}35)$$
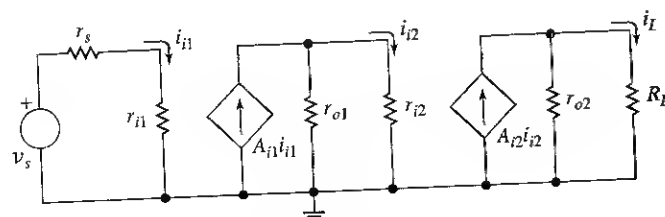

FIGURE 6–20 Two amplifier stages with interstage loading in the form of current division

To obtain an equation for the overall voltage gain, it suffices to recognize that the output voltage is the product of i_L and R_L, which yields

$$\frac{v_L}{v_S} = \frac{A_{i1} A_{i2} R_L}{r_S + r_{i1}} \left(\frac{r_{o1}}{r_{o1} + r_{i2}}\right)\left(\frac{r_{o2}}{r_{o2} + R_L}\right) \quad (6\text{-}36)$$

The last two terms in equation 6–36 are the two current division fractions. If both output resistances approach infinity, which is the case for ideal amplifiers, then both current division fractions will approach unity. Therefore, the first term in equation 6–36 represents the overall voltage gain under ideal conditions.

Invoking the general form $A_v = A_i(R_L/r_i)$ we can easily establish in equation 6–36 that the overall current gain must be

$$A_i(\text{ovr}) = A_{i1} A_{i2}\left(\frac{r_{o1}}{r_{o1} + r_{i2}}\right)\left(\frac{r_{o2}}{r_{o2} + R_L}\right) \quad (6\text{-}37)$$

Note also that $r_S + r_{i1}$ would be the input resistance immediately following v_S, even though r_S is the internal resistance of the source v_S. Therefore, if we remove the r_S term in equation 6–36, the resulting expression would be the voltage gain v_L/v_i, referenced to the voltage across the input terminals of the amplifier.

SUMMARY

This chapter has presented the basics of the amplification function in general terms, that is, without regard to the actual devices employed in the construction of amplifiers. At the end of this chapter, the student should be able to understand the following concepts:

- Amplifiers are used for boosting electrical signals.
- Amplifiers are characterized by a set of functional parameters.
- Amplifiers can be analyzed through equivalent electric circuits called models.
- There are different approaches for the modeling of amplifiers.
- The amplification of signals can be inverting or noninverting.
- Amplifiers can be single-ended or differential.
- Differential amplifiers can reduce noise and interference.
- Two or more amplifiers can be cascaded to form a multistage amplifier.

EXERCISES

SECTION 6–2

Amplifier Characteristics

6–1. An ac amplifier has a voltage gain of 55 and a power gain of 456.5. The ac output current is 24.9 mA rms and the ac input resistance is 200 Ω. Find

   (a) the current gain,

   (b) the rms value of the ac input current,

   (c) the rms value of the ac input voltage,

   (d) the rms value of the ac output voltage,

   (e) the ac output resistance, and

   (f) the output power.

6–2. An ac amplifier has a current gain of 0.95 and a voltage gain of 100. The ac input voltage is 120 mV rms and the ac input resistance is 25 Ω. Find the output power.

6–3. The signal source connected to the input of an ac amplifier has an internal resistance of 1.2 kΩ. The voltage gain of the amplifier from its input to its output is 140. What minimum value of input resistance should the amplifier have if the voltage gain from signal source to amplifier output is to be at least 100?

6-4. The amplifier in Figure 6-21 has current gain $A_i = 80$ from amplifier input to amplifier output. Find the load current i_L.

6-5. An ac amplifier is driven by a 20-mV-rms signal source having internal resistance 1 kΩ. The output resistance of the amplifier is 50 Ω. The voltage gain of the amplifier from its input to its (open-circuit) output (A_v) is 150. What power is delivered to the load if the amplifier is matched to its source and matched to its source and load?

SECTION 6-3
Amplifier Models

6-6. Using the model of Figure 6-3 and placing a short circuit across the output terminals, show that the short-circuit current gain, A_{isc} (current entering output terminal), is given by

$$A_{isc} = \frac{-A_v r_i}{r_o}$$

6-7. An amplifier modeled as in Figure 6-4 has $r_i = 5$ kΩ, $A_i = 130$, and $r_o = 90$ kΩ. A current source i_s with internal resistance of 15 kΩ is connected to the input of the amplifier, and a load resistance of 2 kΩ is connected across the output terminals. If $i_s = 1$ mA rms, what is the voltage across and the current through the load resistance?

6-8. An amplifier modeled as in Figure 6-11 has input resistance of 3 kΩ, its transconductance is 55 mS, and the output resist-

ance is 90 kΩ. If $v_S = 60$ mV rms, $r_S = 600$ Ω, and $R_L = 1.5$ kΩ, find the power delivered to the load.

SECTION 6-4
Multistage Amplifiers

6-9. The in-circuit voltage gains of the stages in a multistage amplifier are shown in Figure 6-22. Find

(a) the overall voltage gain, v_o/v_{in}; and

(b) the voltage gain that would be necessary in a fifth stage, which, if added to the cascade, would make the overall voltage gain 10^5.

6-10. The in-circuit voltage gains of the stages in a multistage amplifier are shown in Figure 6-22. (The gain of the first stage does not include loading by the signal source and that of the fourth stage does not include loading by a load resistor.) The input resistance to the first stage is 20 kΩ, and the output resistance of the fourth stage is 20 Ω. The amplifier is driven by a signal source having resistance 25 kΩ, and a 12-Ω load is connected to the output of the fourth stage. If the source voltage is $v_s = 5$ mV rms, find

(a) the load voltage, v_L;

(b) the power gain, between the input to the first stage and the load.

6-11. It is desired to construct a three-stage amplifier whose overall voltage gain is 500. The in-circuit voltage gains of the first two stages are to be equal, and the voltage
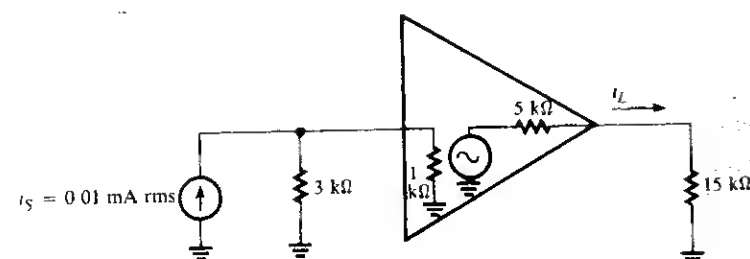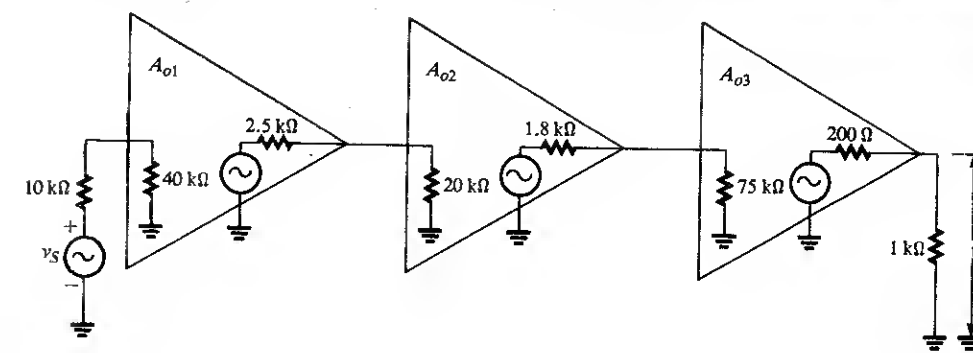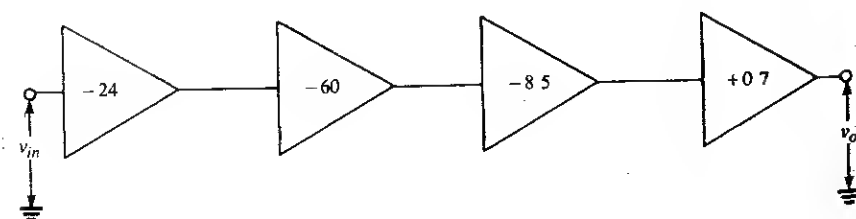
gain of the third stage is to be one-half that of each of the first two. What should be the voltage gain of each stage?

6-12. The open-circuit (unloaded) voltage gains of the stages in the multistage

amplifier shown in Figure 6-23 are $A_{o1} = -42$, $A_{o2} = -26$, and $A_{o3} = 1.8$. Find the overall voltage gain v_L/v_S.

FIGURE 6-23　(Exercise 6-12)

FIGURE 6-21　(Exercise 6-4)

FIGURE 6-22　(Exercises 6-9 and 6-10)